MATHEMATICS

Class-IX

Topic-1

NUMBER SYSTEM

INDEX		
S. No.	**Topic**	**Page No.**

CH-01
NUMBER SYSTEM

A. INTRODUCTION TO NUMBER SYSTEM & RATIONAL NUMBERS

(a) Classification of Numbers

(i) Natural numbers : Counting numbers are known as **natural numbers**.
N = { 1, 2, 3, 4, ... }.

(ii) Whole numbers : All natural numbers together with 0 form the collection of all **whole numbers**.
W = { 0, 1, 2, 3, 4, ... }.

(iii) Integers : All whole numbers and negative of natural numbers form the collection of all **integers**.
I or **Z** = { ..., – 3, – 2, – 1, 0, 1, 2, 3, ... }.

(iv) Rational numbers : The numbers which can be expressed in the form of $\frac{p}{q}$, where **p** and **q** are integers and $q \neq 0$.

For example : $\frac{2}{3}, -\frac{37}{15}$.

❖ All natural numbers, whole numbers and integers are rational.

(v) Real numbers : Numbers which can represent actual physical quantities in a meaningful way are known as **real numbers**. They can be represented on the number line.

❖ Real numbers include all rational and irrational numbers.

(vi) Prime numbers : Prime numbers are natural numbers greater than 1 and each of which is divisible by 1 and itself only. For example : 2, 3, 5, 7, 11, 13, 17, 19, 23, ... etc.

(vii) Composite numbers : All natural numbers greater than 1 which are not prime numbers.

❖ 1 is neither prime nor composite number.

(viii) Co-prime Numbers : If the H.C.F. of the given numbers (not necessarily prime) is 1 then they are known as **co-prime numbers**. For example : 4, 9 are co-prime as H.C.F. of (4, 9) = 1.

❖ Any two consecutive numbers will always be co-prime.

(ix) Even Numbers : Integers divisible by 2
E = { ..., – 2, 0, 2, ... }.
(x) Odd Numbers : Integers not divisible by 2
O = { ..., – 3, –1, 1, 3, ... }.

(b) Rational number in decimal form

(i) Terminating Decimal :
Let **x** be a rational number whose decimal expansion terminates. Then, **x** can be expressed in the form $\frac{p}{q}$, where **p** and **q** are co-prime, and prime factorization of **q** is of the form $2^m \times 5^n$, where **m**, **n** are non-negative integers. In such rational numbers finite decimal number of digit occurs after decimal.

For example : $\frac{1}{2} = 0.5$, $\frac{11}{16} = 0.6875$, $\frac{3}{20} = 0.15$ etc.

(ii) Non-Terminating and Repeating (Recurring Decimal) :

Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of **q** is not of the form $2^m \times 5^n$, where **m, n** are non - negative integers. Then, **x** has a decimal expansion which is non - terminating repeating. In this a set of digits or a digit is repeated continuously.

For example : $\frac{2}{3} = 0.6666..... = 0.\overline{6}$ and $\frac{5}{11} = 0.454545..... = 0.\overline{45}$.

(c) Representation of rational number on a real number line

❖ **Representing terminating Decimals on Number line :**

The process of visualization of number on the number line through a magnifying glass is known as **successive magnification.**

Sometimes, we are unable to check the numbers like 3.765 and $4.\overline{26}$ on the number line. We seek the help of magnifying glass by dividing the part into subparts and subparts into again equal subparts to ensure the accuracy of the given number.

❖ **Method to Find Such Numbers on the Number Line**

1. Choose the two consecutive integral numbers in which the given number lies.

2. Choose the two consecutive decimal points in which the given decimal part lies by dividing the two given decimal parts into required equal parts.

3. Visualize the required number through magnifying glass.

(d) Conversion of recuring decimal into fraction

(i) Long Method :

Step 1 : Take the mixed recurring decimal and let it be equal to **x.**

Step 2 : Count the number of nonrecurring digits after the decimal point. Let it be **n**.

Step 3 : Multiply both sides of equation by $\mathbf{10^n}$ so that only the repeating decimal is on the right hand side of the decimal point.

Step 4 : Multiply both sides of equation obtained in step 3 by $\mathbf{10^m}$ where **m** is the number of repeating digits in the decimal part.

Step 5 : Subtract the equation in **step 3** from equation obtained in **step 4**.

Step 6 : Divide both sides of the resulting equation by the coefficient of **x**.

Step 7 : Write the rational number thus obtained in the simplest $\frac{p}{q}$ form.

(ii) Direct Method :

Step1 : To obtain numerator subtract the number formed by non-repeating digits from the complete number without decimal. (Consider repeated digits only once.)

Step2 : To obtain denominator take number of nines = Number of repeating digits & after that put number of zeros = number of non-repeating digits.

(e) Finding Rational Numbers Between Two Integral Number :

Method - I

Let a & b are two given rational numbers such that a < b.
If n rational numbers are inserted between a & b.

Then, multiply numerator and denominator of a and b by $\frac{n+1}{n+1}$.

$a = a \times \frac{n+1}{n+1}$ and $b = b \times \frac{n+1}{n+1}$.

Then, as we increase the value of numerator we get rational numbers between a & b.

Method - II

Let a & b are two given rational numbers such that a < b then, a < b

$\Rightarrow \quad a + a < b + a$ [adding a both sides]

$\Rightarrow \quad 2a < a + b \quad \Rightarrow \quad a < \frac{a+b}{2}$

Again, $a < b$

$\Rightarrow$ $a + b < b + b.$ [adding b both sides]

$\Rightarrow$ $a + b < 2b$ $\Rightarrow$ $\frac{a+b}{2} < b.$

$\therefore$ $a < \frac{a+b}{2} < b.$ i.e. $\frac{a+b}{2}$ lies between a and b.

Hence 1st rational number between a and b is $\frac{a+b}{2}$.

For next rational number $\frac{a+\frac{a+b}{2}}{2} = \frac{\frac{2a+a+b}{2}}{2} = \frac{3a+b}{4}$

$\therefore$ $a < \frac{3a+b}{4} < \frac{a+b}{2} < b.$

next $\frac{\frac{a+b}{2}+b}{2} = \frac{a+b+2b}{2\times 2} = \frac{a+3b}{4}$

$\therefore$ $a < \frac{3a+b}{4} < \frac{a+b}{2} < \frac{a+3b}{4} < b.$ and continues like this.

Solved Examples

Example.1

Is (39, 93) a coprime ?

Sol. HCF of (39, 93) is 3.

$\therefore$ (39, 93) is not coprime.

Example.2

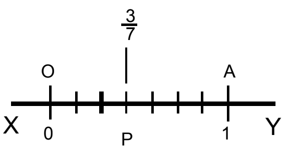
Represent $\frac{3}{7}$ on a real number line.

Sol. **(i)** Draw a line XY which extends endlessly in both the directions.

(ii) Take a point O on it and let it represent O (zero).

(iii) Taking the fixed length, called unit length, mark off OA = 1 unit, as shown in figure below

(iv) Divide OA into 7 equal parts. OP represents $\frac{3}{7}$ of a unit.



Example.3

Represent $\frac{7}{5}$ on a real number line.

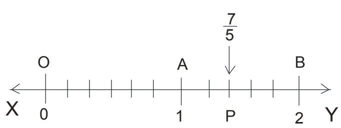
Sol. $\frac{7}{5} = 1\frac{2}{5}$

(i) Draw a line XY which extends endlessly in both the directions.

(ii) Take a point O on it and let it represent 0 (zero).

(iii) Taking the fixed length, called unit length, mark off OA = 1 unit and OB = 2 unit.

(iv) Divide OA and AB into 5 equal parts. OP represents the rational number $\frac{7}{5}$.



Example.4

Represent $-\frac{13}{4}$ on a real number line.

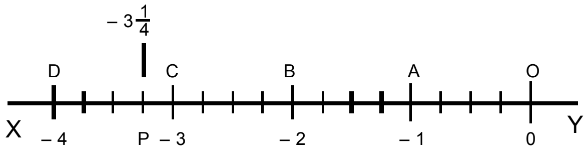
Sol. $-\frac{13}{4} = -3\frac{1}{4}$

(i) Draw a line XY which extends endlessly in both the directions.

(ii) Take a point O on it and let it represent 0 (zero).

(iii) Taking the fixed length, called unit length, mark off OA = 1 unit and OB = 2 unit and OC = 3 unit on the left side of O.

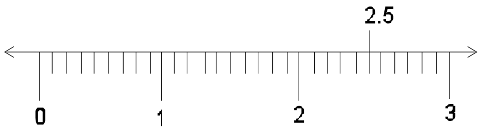
(iv) Divide OA , AB, BC and CD into 4 equal parts. OP represents the rational number $-\frac{13}{4}$ of a unit.



Example.5

Represent 2.5 on a real number line.

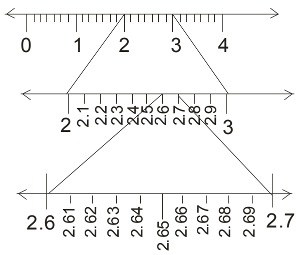
Sol.



Example.6

Represent 2.65 on a real number line by process of magnification.

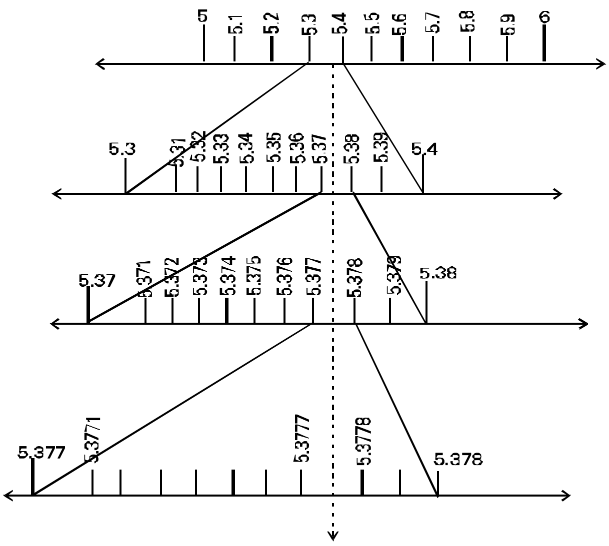
Sol.



Example.7

Visualize the representation of $5.3\overline{7}$ on the number line upto four decimal places.

Sol.



Example.8

Express $0.\overline{6}$ to $\frac{p}{q}$ form.

Sol. Let $x = 0.\overline{6}$

i.e. $x = 0.6666.....$ ...(i)

Multiply both sides of eq.(i) by 10.

$10x = 6.666.....$ ...(ii)

Subtract eq.(i) from eq.(ii)

$$\begin{aligned} 10x &= 6.666.... \\ -x &= -0.666.... \\ \hline 9x &= 6 \end{aligned}$$

$$x = \frac{6}{9}$$

$$x = \frac{2}{3}.$$

Example.9

Express $0.\overline{47}$ to $\frac{p}{q}$ form.

Sol. Let $x = 0.\overline{47}$

i.e. $x = 0.474747.....$ ...(i)

Multiply both sides of eq.(i) by 100.

$100x = 47.474747.....$ ...(ii)

Subtract eq.(i) from eq.(ii)

$$\begin{aligned} 100x &= 47.474747.... \\ -x &= -0.474747.... \\ \hline 99x &= 47 \end{aligned}$$

$$x = \frac{47}{99}.$$

Example.10

Express $0.12\overline{3}$ to $\frac{p}{q}$ form.

Sol. Let $x = 0.12\overline{3}$

i.e. $x = 0.12333.....$ ...(i)

Multiply both sides of eq.(i) by 100.

$100x = 12.333.....$ ...(ii)

Multiply both sides of eq.(ii) by 10

$1000x = 123.333.....$ ...(iii)

Subtract eq.(ii) from eq.(iii)

$$\begin{aligned} 1000\,x &= 123.333.... \\ -100\,x &= -12.333.... \\ \hline 900\,x &= 111.000 \end{aligned}$$

$$x = \frac{111}{900} \Rightarrow x = \frac{3 \times 37}{900} = \frac{37}{300}.$$

Example.11

Express the following to $\frac{p}{q}$ form using direct method :

(i) $0.\overline{45}$ **(ii)** $0.7\overline{37}$ **(iii)** $0.46\overline{573}$

Sol. **(i)** $0.\overline{45} = \frac{45-0}{99} = \frac{45}{99} = \frac{5}{11}$ **(ii)** $0.7\overline{37} = \frac{737-7}{990} = \frac{730}{990} = \frac{73}{99}$.

(iii) $0.46\overline{573} = \frac{46573-46}{99900} = \frac{46527}{99900}$.

Example.12

Find 4 rational numbers between 2 and 3.

Sol. **Steps :**

(i) Multiplying 2 and 3 in N^r and D^r with (4+1).

(ii) $2 = \frac{2\times(4+1)}{(4+1)} = \frac{10}{5}$ & $\frac{3\times(4+1)}{(4+1)} = \frac{15}{5}$.

(iii) So, the four required numbers are $\frac{11}{5}, \frac{12}{5}, \frac{13}{5}, \frac{14}{5}$.

Example.13

Find 3 rational numbers between $\frac{1}{3}$ & $\frac{1}{2}$.

Sol. $\frac{\frac{1}{3}+\frac{1}{2}}{2} = \frac{\frac{2+3}{6}}{2} = \frac{5}{12}$

$\therefore$ $\frac{1}{3}, \frac{5}{12}, \frac{1}{2}$

$\frac{\frac{1}{3}+\frac{5}{12}}{2} = \frac{\frac{4+5}{12}}{2} = \frac{9}{24}$

$\therefore$ $\frac{1}{3}, \frac{9}{24}, \frac{5}{12}, \frac{1}{2}$

$\frac{\frac{5}{12}+\frac{1}{2}}{2} = \frac{\frac{5}{12}+\frac{6}{12}}{2} = \frac{11}{24}$

$\therefore$ $\frac{1}{3}, \frac{9}{24}, \frac{5}{12}, \frac{11}{24}, \frac{1}{2}$.

So, the required 3 rational number are $\frac{9}{24}, \frac{5}{12}, \frac{11}{24}$

Check Your Level

1. Represent the number $\frac{3}{5}$ on the number line.
2. Find a fraction between $\frac{3}{8}$ and $\frac{2}{5}$.
3. Insert 5 rational numbers between 3 and 4.
4. Which of the following fractions yield a recurring decimal ?
   $\frac{5}{3}; \frac{7}{16}; \frac{9}{14}; \frac{5}{7}; \frac{12}{5}; \frac{6}{11}$
5. Represent 1. 129129129…… as a fraction.

Answers

2. $\frac{31}{80}$ **3.** $\frac{19}{6}, \frac{20}{6}, \frac{21}{6}, \frac{22}{6}, \frac{23}{6}$ **4.** $\frac{5}{3}, \frac{9}{14}, \frac{5}{7}, \frac{6}{11}$

5. $x = \frac{1128}{999}$.

B. IRRATIONAL NUMBERS

All real number which are not rational are called irrational numbers. These are non-recurring as well as non-terminating type of decimal numbers.

i.e. $\sqrt{2}$, $\sqrt[3]{4}$, $2+\sqrt{3}$, $\sqrt{2+\sqrt{3}}$, $\sqrt[4]{\sqrt[7]{3}}$, π, etc..

(a) Proof of irrationality of numbers

To prove the irrationality of a given number, a process is done by contradiction method. In logic, proof by contradiction is a form of proof, and more specifically a form of indirect proof, that establishes the truth or validity of a proposition. It starts by assuming that the opposite proposition is true, and then shows that such an assumption lead to a contradiction.

(b) Insertion of irrational numbers between two real numbers

Let a and b are two given real numbers, then irrational number between a and b is $\sqrt{a \times b}$ Provide $a \times b$ is not a perfect square.

(c) Irrational Number on a Number Line

Irrational Number in Decimal Form :

$\sqrt{2}$ = 1.414213............... i.e. it is non-recurring as well as non-terminating.

$\sqrt{3}$ = 1.732050807.......... i.e. it is non-recurring as well as non-terminating.

Properties of Irrational Number :

(i) Negative of an irrational number is an irrational number. **e.g.** $-\sqrt{3}$, $-\sqrt[4]{5}$ are irrational.

(ii) Sum and difference of a rational and an irrational number is always an irrational number.

(iii) Sum, product and difference of two irrational numbers is either rational or irrational number.

(iv) Product of a rational number with an irrational number is either rational or irrational.

(d) Geometrical representation of real numbers

To represent any real number $\sqrt{x}$ on number line we follow the following steps :

STEP I : Obtain the positive real number **x** (say).

STEP II : Draw a line and mark a point A on it.

STEP III : Mark a point B on the line such that AB = x units.

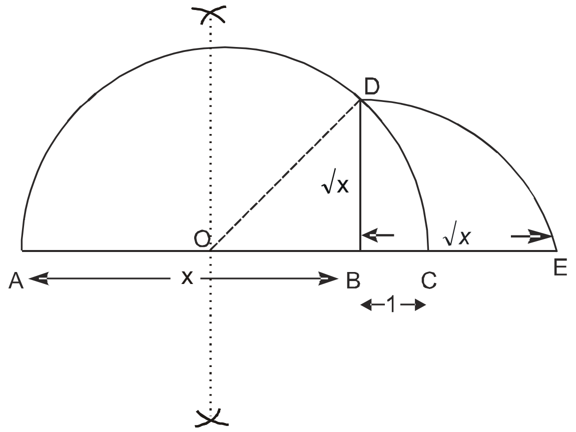
STEP IV : From point B mark a distance of 1 unit and mark the new point as C. Such that ABC is a straight line.

STEP V : Find the mid-point of AC by drawing the perpendicular bisector of line segment AC and mark the point as O.

STEP VI : Draw a semi circle with centre O and radius OC.

STEP VII : Draw a line perpendicular to AC passing through B and intersecting the semi circle at D. Length BD is equal to $\sqrt{x}$.

STEP VIII : Taking B as centre and BD as radius, draw an arc cutting OC produced at E. Distance BE represents $\sqrt{x}$.



EXPLANATION :

We have,

AB = x units and BC = 1 unit.

$\therefore$ AC = (x + 1) units

$\Rightarrow$ OA = OC = $\frac{x+1}{2}$ units

$\Rightarrow$ OD = $\frac{x+1}{2}$ units [$\because$ OA = OC = OD]

Now, OB = AB – OA = $x - \frac{x+1}{2} = \frac{x-1}{2}$

Using Pythagoras Theorem in ΔOBD, we have

$OD^2 = OB^2 + BD^2 \Rightarrow BD^2 = OD^2 - OB^2$

$\Rightarrow BD^2 = \left(\frac{x+1}{2}\right)^2 - \left(\frac{x-1}{2}\right)^2 \Rightarrow BD = \sqrt{\frac{(x^2+2x+1) - (x^2-2x+1)}{4}}$

$\Rightarrow BD = \sqrt{\frac{4x}{4}} \Rightarrow BD = \sqrt{x}$

This shows that $\sqrt{x}$ exists for all real numbers x > 0.

Solved Examples

Example. 14

Prove that $\sqrt{2}$ is an irrational number.

Sol. Let assume on the contrary that $\sqrt{2}$ is a rational number.

Then, there exists positive integer a and b such that $\sqrt{2} = \frac{a}{b}$ where, a and b are coprime i.e. their HCF is1.

$\Rightarrow (\sqrt{2})^2 = \left(\frac{a}{b}\right)^2 \Rightarrow 2 = \frac{a^2}{b^2}$

$\Rightarrow a^2 = 2b^2 \Rightarrow a^2$ is a multiple of 2

$\Rightarrow$ a is a multiple of 2 ...(i)

a = 2c for some integer c.

$\Rightarrow a^2 = 4c^2 \Rightarrow 2b^2 = 4c^2$

$\Rightarrow b^2 = 2c^2 \Rightarrow b^2$ is a multiple of 2

$\Rightarrow$ b is a multiple of 2 ....(ii)

From (i) and (ii), a and b have at least 2 as a common factor. But this contradicts the fact that a and b are co-prime. This means that $\sqrt{2}$ is an irrational number.

Example.15

Prove that $3 - \sqrt{5}$ is an irrational number.

Sol. Let assume that on the contrary that $3 - \sqrt{5}$ is rational.

Then, there exist co-prime positive integers a and b such that,

$$3 - \sqrt{5} = \frac{a}{b} \quad \Rightarrow \quad 3 - \frac{a}{b} = \sqrt{5}$$

$$\Rightarrow \quad \frac{3b-a}{b} = \sqrt{5} \quad \Rightarrow \quad \sqrt{5} \text{ is rational } \left[\text{a, b are integer } \therefore \frac{3b-a}{b} \text{ is a rational number}\right]$$

This contradicts the fact that $\sqrt{5}$ is an irrational number.

Hence, $3 - \sqrt{5}$ is an irrational number.

Example.16

Insert an irrational number between 2 and 3.

Sol. $\sqrt{2 \times 3} = \sqrt{6}$.

Example.17

Find two irrational number between 2 and 2.5.

Sol. **1st Method :** $\sqrt{2 \times 2.5} = \sqrt{5}$

Since, there is no rational number whose square is 5. So, $\sqrt{5}$ is an irrational number.

Also, $\sqrt{2 \times \sqrt{5}}$ is an irrational number.

2nd Method : 2.101001000100001............ is between 2 and 5 and it is non-recurring as well as non-terminating.

Also, 2.201001000100001 .... and so on.

Example.18

Plot $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ on a number line.

Sol. Let X'OX be a horizontal line, taken as the x - axis and let O be the origin. Let O represents 0 (zero).

Take OA = 1 unit and draw AB $\perp$ OA such that AB = 1 unit.

Join OB. Then,

$OB = \sqrt{OA^2 + AB^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$ units.

With O as centre and OB as radius, draw an arc, meeting OY at P.

Then, OP = OB = $\sqrt{2}$ units.

Thus the point P represents $\sqrt{2}$ on the real line.

Now draw BC $\perp$ OB such that BC = 1 unit.

Join OC. Then,

$OC = \sqrt{OB^2 + BC^2} = \sqrt{\left(\sqrt{2}\right)^2 + 1^2} = \sqrt{3}$ units.

With O as centre and OC as radius, draw an arc, meeting OY at Q.

Then, OQ = OC = $\sqrt{3}$ units.

Thus the point Q represents $\sqrt{3}$ on the real line.

Now draw CD $\perp$ OC such that CD = 1 unit.

Join OD. Then,

$OD = \sqrt{OC^2 + CD^2} = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = \sqrt{4} = 2$ units.

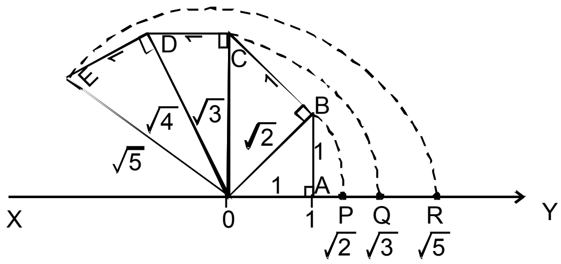
Now draw DE $\perp$ OD such that DE = 1 unit.

Join OE. Then,

OE = $\sqrt{OD^2 + DE^2}$ = $\sqrt{(2)^2 + 1^2}$ = $\sqrt{5}$ units.

With O as centre and OE as radius, draw an arc, meeting OY at R.

Then, OR = OE = $\sqrt{5}$ units.



❖ **Another Method for :**

(i) Plot $\sqrt{2}$, $\sqrt{3}$

Draw a number line and mark a point O, representing zero, on it. Suppose a point A represents 1. Then

OA = 1. Now draw a right triangle OAB such that AB = OA = 1.

By pythagoras theorem,

OB = $\sqrt{OA^2 + AB^2}$ = $\sqrt{1^2 + 1^2}$ = $\sqrt{2}$ units.

Now, draw an arc with centre O and radius OB. It cuts the number line at C.

Then, OC = OB = $\sqrt{2}$ units.

Thus the point C represents $\sqrt{2}$ on the real line.

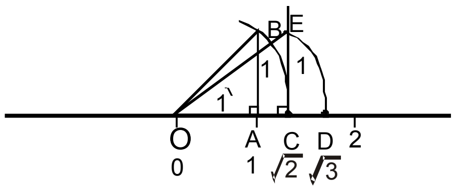
Now, draw a right triangle OEC such that CE = AB = 1 unit.

Again by pythagoras theorem,

OE = $\sqrt{OC^2 + CE^2}$ = $\sqrt{(\sqrt{2})^2 + 1^2}$ = $\sqrt{3}$ units.

Now, draw an arc with centre O and radius OE. It cuts the number line at D.

Then, OD = OE = $\sqrt{3}$ units.



Check Your Level

1. Which of the following numbers are not rational?
1.256; 0.45454545…; 0.05005000500005 …; 5.51551555151…; 2.012340123401234…;

2. Find two irrational numbers between $\sqrt{5}$ and $\sqrt{6}$.

3. Prove that $\sqrt{3}$ is irrational number.

4. Represent $\sqrt{6}$ on the number line.

5. Represent $\sqrt{7.3}$ on the number line.

Answers

1. 0.05005000500005 …; 5.51551555151…; **2.** $\sqrt{5.1}$, $\sqrt{5.2}$

(C) SURDS AND THEIR APPLICATION

(a) Surds

An irrational number of the form $\sqrt[n]{a}$ is given a special name **Surd**, where **'a'** is called **radicand** and it should always be a rational number. Also the symbol $\sqrt[n]{\ }$ is called the **radical sign** and the index **n** is called **order** of the surd. $\sqrt[n]{a}$ is read as **'nth root of a'** and can also be written as $a^{\frac{1}{n}}$.

(b) Law of Surds

(i) $\left(\sqrt[n]{a}\right)^n = \sqrt[n]{a^n} = a$ **(ii)** $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$

(iii) $\sqrt[n]{a} \div \sqrt[n]{b} = \sqrt[n]{\frac{a}{b}}$ **(iv)** $\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a} = \sqrt[m]{\sqrt[n]{a}}$

(v) $\sqrt[n]{a} = \sqrt[n\times p]{a^p}$ or $\sqrt[n]{a^m} = \sqrt[n\times p]{a^{m\times p}}$

(c) Operation on Surds

(i) Addition and Subtraction of Surds :

Addition and subtraction of surds are possible only when order and radicand are same i.e. only for like surds.

The addition of surds follow the following rules. Summation of same degree surds is distributive.

$a\sqrt[n]{p} + b\sqrt[n]{p} = (a+b)\sqrt[n]{p}$

The subtraction of surds follow the following rules. Subtraction of same degree surds is distributive.

$a\sqrt[n]{p} - b\sqrt[n]{p} = (a-b)\sqrt[n]{p}$

(ii) Multiplication and Division of Surds :

For multiplication and division we have to check the order if it is not same then first we make the order of surd same by using LCM of indices. Then we follow the following rule

$a\sqrt[n]{p} \times b\sqrt[n]{q} = (a \times b)\sqrt[n]{p\times q}$

$$\frac{a\sqrt[n]{p}}{b\sqrt[n]{q}} = \left(\frac{a}{b}\right)\sqrt[n]{\left(\frac{p}{q}\right)}$$

(iii) Comparison of Surds :

It is clear that if x > y > 0 and n > 1 is a positive integer then $\sqrt[n]{x} > \sqrt[n]{y}$.

(d) Rationalization of Surds

Rationalizing factor : Product of two surds is a rational number then each of them is called the **rationalizing factor (R.F.)** of the other. The process of converting a surd to a rational number by using an appropriate multiplier is known as **rationalization.**

When the denominator of an expression contains a term with a square root (or a number with radical sign), the process of converting it to an equivalent expression whose denominator is a rational number is called **rationalizing** the **denominator**.

❖ Rationalizing factor of $a^{\frac{1}{n}}$ is $a^{1-\frac{1}{n}}$ where a is a real number.

Solved Examples

Example.19

$\sqrt{75}-\sqrt{45}+\sqrt{50}-\sqrt{32}$

Sol. $\sqrt{75}-\sqrt{45}+\sqrt{50}-\sqrt{32}$

$= 5\sqrt{3}-3\sqrt{5}+5\sqrt{2}-4\sqrt{2}$

$= 5\sqrt{3}-3\sqrt{5}+\sqrt{2}$.

Example.20

Simplify : $5\sqrt[3]{250}+7\sqrt[3]{16}-14\sqrt[3]{54}$

Sol. $5\sqrt[3]{250}+7\sqrt[3]{16}-14\sqrt[3]{54} = 5\sqrt[3]{125\times 2}+7\sqrt[3]{8\times 2}-14\sqrt[3]{27\times 2}$

$= 5\times 5\sqrt[3]{2}+7\times 2\sqrt[3]{2}-14\times 3\times\sqrt[3]{2} = (25+14-42)\sqrt[3]{2}$ **= –** $3\sqrt[3]{2}$.

Example.21

Simplify : $\sqrt[3]{2}\times\sqrt[4]{3}$.

Sol. $\sqrt[3]{2}\times\sqrt[4]{3}$

$= \sqrt[12]{2^4}\times\sqrt[12]{3^3}$ [order should be made same]

$= \sqrt[12]{2^4\times 3^3} = \sqrt[12]{16\times 27} = \sqrt[12]{432}$.

Example.22

Simplify : $\sqrt{8a^5b} \times \sqrt[3]{4a^2b^2}$

Sol. $\sqrt{8a^5b} \times \sqrt[3]{4a^2b^2} = \sqrt[6]{8^3a^{15}b^3}\times\sqrt[6]{4^2a^4b^4}$

$= \sqrt[6]{2^{13}a^{19}b^7} = 2^2a^3 \; b \; \sqrt[6]{2ab} = 4a^3 \; b \; \sqrt[6]{2ab}$.

Example.23

Divide : $\sqrt{24} \div \sqrt[3]{200}$.

Sol. $\sqrt{24} \div \sqrt[3]{200} = \dfrac{\sqrt{24}}{\sqrt[3]{200}} = \dfrac{\sqrt[6]{(24)^3}}{\sqrt[6]{(200)^2}} = \sqrt[6]{\dfrac{216}{625}}$.

Example.24

Which is greater :

(i) $\sqrt[3]{6}$ and $\sqrt[5]{8}$ **(ii)** $\sqrt{\dfrac{1}{2}}$ and $\sqrt[3]{\dfrac{1}{3}}$

Sol. (i) $\sqrt[3]{6}$ and $\sqrt[5]{8}$

L.C.M. of 3 and 5 is 15.

$\sqrt[3]{6} = \sqrt[3\times 5]{6^5} = \sqrt[15]{7776}$

$\sqrt[5]{8} = \sqrt[3\times 5]{8^3} = \sqrt[15]{512}$

$\therefore \sqrt[15]{7776} > \sqrt[15]{512} \Rightarrow \sqrt[3]{6} > \sqrt[5]{8}$.

(ii) $\sqrt{\dfrac{1}{2}}$ and $\sqrt[3]{\dfrac{1}{3}}$

L.C.M. of 2 and 3 is 6.

$\sqrt[6]{\left(\dfrac{1}{2}\right)^3}$ and $\sqrt[6]{\left(\dfrac{1}{3}\right)^2}$

$\sqrt[6]{\frac{1}{8}}$ and $\sqrt[6]{\frac{1}{9}}$ $\left[\text{As } 8 < 9 \ \therefore \frac{1}{8} > \frac{1}{9}\right]$

So, $\sqrt[6]{\frac{1}{8}} > \sqrt[6]{\frac{1}{9}} \Rightarrow \sqrt{\frac{1}{2}} > \sqrt[3]{\frac{1}{3}}$.

Example.25

Arrange $\sqrt{2}$, $\sqrt[3]{3}$ and $\sqrt[4]{5}$ in ascending order.

Sol. $\sqrt{2}$, $\sqrt[3]{3}$ and $\sqrt[4]{5}$

L.C.M. of 2, 3, 4 is 12.

$\therefore \quad \sqrt{2} = \sqrt[2\times 6]{2^6} = \sqrt[12]{64}$

$\sqrt[3]{3} = \sqrt[3\times 4]{3^4} = \sqrt[12]{81}$

$\sqrt[4]{5} = \sqrt[4\times 3]{5^3} = \sqrt[12]{125}$

As, 64 < 81 < 125.

$\therefore \quad \sqrt[12]{64} < \sqrt[12]{81} < \sqrt[12]{125} \Rightarrow \sqrt{2} < \sqrt[3]{3} < \sqrt[4]{5}$.

Example.26

Rationalize the denominator $\frac{1}{\sqrt{162}}$.

Sol. $\frac{1}{\sqrt{162}} = \frac{1}{\sqrt{81\times 2}}$

$= \frac{1}{9\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{18}$.

❖ Rationalising factor of a + b $\sqrt{c}$ is a – b $\sqrt{c}$ where a,b,c are rational numbers.

Example.27

Rationalize the denominator $\frac{1}{7+5\sqrt{3}}$.

Sol. $\frac{1}{7+5\sqrt{3}} = \frac{1}{7+5\sqrt{3}} \times \frac{7-5\sqrt{3}}{7-5\sqrt{3}}$

$= \frac{7-5\sqrt{3}}{49-75} = \frac{7-5\sqrt{3}}{-26} = \frac{5\sqrt{3}-7}{26}$.

Example.28

Rationalize the denominator of $\frac{a^2}{\sqrt{a^2+b^2}+b}$.

Sol. $\frac{a^2}{\sqrt{a^2+b^2}+b} \times \frac{\sqrt{a^2+b^2}-b}{\sqrt{a^2+b^2}-b} = \frac{a^2\left(\sqrt{a^2+b^2}-b\right)}{\left(\sqrt{a^2+b^2}\right)^2-(b)^2}$

$= \frac{a^2\left(\sqrt{a^2+b^2}-b\right)}{a^2+b^2-b^2} = \left(\sqrt{a^2+b^2}-b\right)$.

Example.29

If $\frac{3+2\sqrt{2}}{3-\sqrt{2}} = a+b\sqrt{2}$, where a and b are rationals in reduced form then, find the values of a and b.

Sol. LHS $\frac{3+2\sqrt{2}}{3-\sqrt{2}} = \frac{(3+2\sqrt{2})\ (3+\sqrt{2})}{(3-\sqrt{2})\ (3+\sqrt{2})}$

$= \frac{9+3\sqrt{2}+6\sqrt{2}+4}{9-2} = \frac{13+9\sqrt{2}}{7} = \frac{13}{7}+\frac{9}{7}\sqrt{2}$

$\because \frac{13}{7}+\frac{9}{7}\sqrt{2} = a+b\sqrt{2}$

Equating the rational and irrational parts

We get $a=\frac{13}{7}$, $b=\frac{9}{7}$.

Example.30

If $x=\frac{1}{2+\sqrt{3}}$, find the value of $x^3 - x^2 - 11x + 3$.

Sol. As, $x=\frac{1}{2+\sqrt{3}} = 2-\sqrt{3}$ $\Rightarrow$ $x - 2 = -\sqrt{3}$

$\Rightarrow$ $(x-2)^2 = \left(-\sqrt{3}\right)^2$ [By squaring both sides]

$\Rightarrow$ $x^2 + 4 - 4x = 3$ $\Rightarrow$ $x^2 - 4x + 1 = 0$

Now, $x^3 - x^2 - 11x + 3 = x(x^2 - 4x + 1) + 3(x^2 - 4x + 1)$

$= x(0) + 3(0) = 0 + 0 = 0.$

Example.31

If $x = 3 - \sqrt{8}$, find the value of $x^3 + \frac{1}{x^3}$.

Sol. $x = 3-\sqrt{8}$

$\therefore$ $\frac{1}{x}=\frac{1}{3-\sqrt{8}}$ $\Rightarrow$ $\frac{1}{x}=3+\sqrt{8}$

Now, $x+\frac{1}{x}=3-\sqrt{8}+3+\sqrt{8}=6$

$\Rightarrow$ $x^3+\frac{1}{x^3}=\left(x+\frac{1}{x}\right)^3-3x\frac{1}{x}\left(x+\frac{1}{x}\right)$ $\Rightarrow$ $x^3+\frac{1}{x^3} = (6)^3 - 3(6)$

$\Rightarrow$ $x^3+\frac{1}{x^3} = 216 - 18$ $\Rightarrow$ $x^3+\frac{1}{x^3} = 198.$

Example.32

If $\sqrt{5} = 2.236$ and $\sqrt{2} = 1.414$, then evaluate : $\frac{3}{\sqrt{5}+\sqrt{2}}+\frac{4}{\sqrt{5}-\sqrt{2}}$

Sol. $\frac{3}{\sqrt{5}+\sqrt{2}}+\frac{4}{\sqrt{5}-\sqrt{2}} = \frac{3\left(\sqrt{5}-\sqrt{2}\right)+4\left(\sqrt{5}+\sqrt{2}\right)}{\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)}$

$= \frac{3\sqrt{5}-3\sqrt{2}+4\sqrt{5}+4\sqrt{2}}{5-2} = \frac{7\sqrt{5}+\sqrt{2}}{5-2}$

$= \frac{7\sqrt{5}+\sqrt{2}}{3}$

$= \frac{7\times 2.236+1.414}{3}$

$= \frac{15.652+1.414}{3}$

$= \frac{17.066}{3} = 5.689$ (approximately)

Check Your Level

1. What is the simplest form of $\sqrt{200} - \sqrt{50}$?

2. Rationalise the denominator of $\frac{5}{\sqrt{10}+\sqrt{5}}$.

3. If x = $\sqrt{2}$ – 1 what is the value of x – 1/x ?

4. Simplify $(\sqrt{5} + 1)^2 + (\sqrt{5} - 1)^2$.

5. If x = $\frac{\sqrt{5}-2}{\sqrt{5}+2}$, find the value of $x + \frac{1}{x}$.

Answers

1. $5\sqrt{2}$ **2.** $\sqrt{10} - \sqrt{5}$ **3.** – 2 **4.** 12 **6.** 18

D. EXPONENTS

(a) Exponents of Real Numbers

(i) Positive Integral Power :

For any real number **a** and a natural number **'n'** we define a^n as :
$a^n = a \times a \times a \times$ ....$\times$ a (n times)

a^n is called the **nth** power of **a**. The real number **'a'** is called the **base** and **'n'** is called the **exponent** of the **nth** power of **a**.
e.g. $2^3 = 2 \times 2 \times 2 = 8$

NOTE :
For any non –zero real number 'a' we define $a^0 = 1$.

e.g.: Thus, $3^o = 1$, 5^o , $\left(\frac{3}{4}\right)^0 = 1$ and so on.

(ii) Negative Integral Power :

For any non–zero real number **'a'** and positive integer **'n'** we define $a^{-n} = \frac{1}{a^n}$.

Thus we have defined a^n for all integral values of **n**, positive, zero or negative. a^n is called the **nth** power of **a**.

(iii) Rational Exponents of a Real number

Principal of nth Root of a Positive Real Numbers :

If **'a'** is a positive real number and **'n'** is a **positive integer**, then the principal **nth root** of **a** is the unique positive real number **x** such that $x^n = a$.
The principal nth root of a positive real number a is denoted by $a^{1/n}$ or $\sqrt[n]{a}$.

❖ **REMARK :**
If **'a'** is negative real number and **'n'** is an **even positive** integer, then the principal **nth** root of **a** is not defined, because an even power of a real number is always positive. Therefore $(-9)^{1/2}$ is a meaningless quantity, if we confine ourselves to the set of real number, only.

(b) Law of Rational Exponents

The following laws hold the rational exponents

(i) $a^m \times a^n = a^{m+n}$ **(ii)** $a^m \;\; a^n = a^{m-n}$

(iii) $(a^m)^n = a^{mn}$ **(iv)** $a^{-n} = \dfrac{1}{a^n}$

(v) $a^{m/n} = (a^m)^{1/n} = (a^{1/n})^m$ i.e. $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$ **(vi)** $(ab)^m = a^m b^m$

(vii) $\left(\dfrac{a}{b}\right)^m = \dfrac{a^m}{b^m}$ **(viii)** $a^{bn} = a^{b+b+b.....n \text{ times}}$

where a, b are positive real numbers and m, n are rational numbers.

Solved Examples

Example.33

Evaluate each of the following :

(i) $5^8 \div 5^3$ **(ii)** $\left(\dfrac{3}{4}\right)^{-3}$

Sol. Using the laws of indices, we have :

(i) $5^8 \div 5^3 = \dfrac{5^8}{5^3} = 5^{8-3} = 5^5 = 3125$. $[\because a^m \;\; a^n = a^{m-n}]$

(ii) $\left(\dfrac{3}{4}\right)^{-3} = \dfrac{1}{\left(\dfrac{3}{4}\right)^3} = \dfrac{1}{\dfrac{3^3}{4^3}} = \dfrac{1}{\dfrac{27}{64}} = \dfrac{64}{27}$ $[\because a^{-n} = \dfrac{1}{a^n}]$

Example.34

Evaluate each of the following :

(i) $\left(\dfrac{1}{2}\right)^5 \times \left(\dfrac{-2}{3}\right)^4 \times \left(\dfrac{3}{5}\right)^{-1}$ **(ii)** $\left(\dfrac{2}{3}\right)^3 \times \left(\dfrac{2}{5}\right)^{-3} \times \left(\dfrac{3}{5}\right)^2$

Sol. (i) We have,

$$\left(\frac{1}{2}\right)^5 \times \left(\frac{-2}{3}\right)^4 \times \left(\frac{3}{5}\right)^{-1} = \left(\frac{1}{2}\right)^5 \times \left(\frac{-2}{3}\right)^4 \times \left(\frac{1}{\frac{3}{5}}\right) = \frac{1^5}{2^5} \times \frac{(-2)^4}{3^4} \times \frac{5}{3}$$

$$= \frac{1 \times 16 \times 5}{32 \times 81 \times 3} = \frac{5}{2 \times 81 \times 3} = \frac{5}{486}.$$

(ii) We have,

$$\left(\frac{2}{3}\right)^3 \times \left(\frac{2}{5}\right)^{-3} \times \left(\frac{3}{5}\right)^2 = \frac{2^3}{3^3} \times \frac{1}{(2/5)^3} \times \frac{3^2}{5^2} = \frac{2^3 \times 5^3 \times 3^2}{3^3 \times 2^3 \times 5^2} = \frac{5}{3}.$$

Example.35

Simplify :

(i) $\dfrac{(25)^{3/2} \times (243)^{3/5}}{(16)^{5/4} \times (8)^{4/3}}$ **(ii)** $\dfrac{16 \times 2^{n+1} - 4 \times 2^n}{16 \times 2^{n+2} - 2 \times 2^{n+2}}$

Sol. We have,

(i) $$\frac{(25)^{3/2} \times (243)^{3/5}}{(16)^{5/4} \times (8)^{4/3}} = \frac{(5^2)^{3/2} \times (3^5)^{3/5}}{(2^4)^{5/4} \times (2^3)^{4/3}} = \frac{5^{2\times3/2} \times 3^{5\times3/5}}{2^{4\times5/4} \times 2^{3\times4/3}} = \frac{5^3 \times 3^3}{2^5 \times 2^4} = \frac{125 \times 27}{32 \times 16} = \frac{3375}{512}$$

(ii) $$\frac{16\times 2^{n+1}-4\times 2^{n}}{16\times 2^{n+2}-2\times 2^{n+2}}=\frac{32\times 2^{n}-4\times 2^{n}}{64\times 2^{n}-8\times 2^{n}}=\frac{2^{n}(32-4)}{2^{n}(64-8)}=\frac{1}{2}.$$

Example.36

Simplify . $\left(\frac{81}{16}\right)^{-3/4}\times\left[\left(\frac{25}{9}\right)^{-3/2}\div\left(\frac{5}{2}\right)^{-3}\right].$

Sol. We have,

$$\left(\frac{81}{16}\right)^{-3/4}\times\left[\left(\frac{25}{9}\right)^{-3/2}\div\left(\frac{5}{2}\right)^{-3}\right]=\left(\frac{3^4}{2^4}\right)^{-3/4}\times\left[\left(\frac{5^2}{3^2}\right)^{-3/2}\div\left(\frac{5}{2}\right)^{-3}\right]$$

$$=\left[\left(\frac{3}{2}\right)^{4}\right]^{-3/4}\times\left[\left(\frac{5}{3}\right)^{2}\right]^{-3/2}\div\left[\left(\frac{5}{2}\right)^{-3}\right]=\left(\frac{3}{2}\right)^{4\times -3/4}\times\left[\left(\frac{5}{3}\right)^{2\times -3/2}\div\left(\frac{5}{2}\right)^{-3}\right]$$

$$=\left(\frac{3}{2}\right)^{-3}\times\left[\left(\frac{5}{3}\right)^{-3}\div\left(\frac{5}{2}\right)^{-3}\right]=\left(\frac{2}{3}\right)^{3}\times\left[\left(\frac{3}{5}\right)^{3}\div\left(\frac{2}{5}\right)^{3}\right]=\frac{2^3}{3^3}\times\left[\frac{3^3}{5^3}\div\frac{2^3}{5^3}\right]=\frac{2^3}{3^3}\times\left[\frac{3^3}{5^3}\times\frac{5^3}{2^3}\right]=1.$$

Example.37

Prove that : $\frac{x^{-1}}{x^{-1}+y^{-1}}+\frac{x^{-1}}{x^{-1}-y^{-1}}=\frac{2y^2}{y^2-x^2}.$

Sol. $$\frac{x^{-1}}{x^{-1}+y^{-1}}+\frac{x^{-1}}{x^{-1}-y^{-1}}=\frac{\frac{1}{x}}{\frac{1}{x}+\frac{1}{y}}+\frac{\frac{1}{x}}{\frac{1}{x}-\frac{1}{y}}=\frac{\frac{1}{x}}{\frac{y+x}{xy}}+\frac{\frac{1}{x}}{\frac{y-x}{xy}}.$$

$$=\frac{xy}{x(y+x)}+\frac{xy}{x(y-x)}=\frac{xy(y-x)+xy(y+x)}{x(y^2-x^2)}$$

$$=\frac{y(y-x)+y(y+x)}{y^2-x^2}=\frac{y^2-xy+y^2+xy}{y^2-x^2}=\frac{2y^2}{y^2-x^2}.$$

Example.38

Find the value of x : $\left(\frac{3}{5}\right)^{x}\left(\frac{5}{3}\right)^{2x}=\frac{125}{27}.$

Sol. $$\left(\frac{3}{5}\right)^{x}\left(\frac{5}{3}\right)^{2x}=\frac{125}{27}$$

$$\left(\frac{5}{3}\right)^{-x}\left(\frac{5}{3}\right)^{2x}=\frac{125}{27}$$

$$\left(\frac{5}{3}\right)^{2x-x}=\frac{125}{27}.$$

$$\left(\frac{5}{3}\right)^{x}=\left(\frac{5}{3}\right)^{3}$$

Because the base is same, so comparing the powers x = 3.

Example.39

If $25^{x-1}=5^{2x-1}-100$, find the value of x.

Sol. We have,

$\Rightarrow\quad 25^{x-1}=5^{2x-1}-100 \qquad \Rightarrow\quad (5^2)^{x-1}=5^{2x-1}-100$

$\Rightarrow\quad 5^{2x-2}-5^{2x-1}=-100 \qquad \Rightarrow\quad 5^{2x-2}-5^{2x-2}.5^{1}=-100$

$\Rightarrow \quad 5^{2x-2}(1-5) = -100 \qquad \Rightarrow \quad 5^{2x-2}(-4) = -100$

$\Rightarrow \quad 5^{2x-2} = 5^2 \qquad \Rightarrow \quad 2x - 2 = 2$

$\Rightarrow \quad 2x = 4 \qquad \Rightarrow \quad x = 2.$

Example.40

Assuming that x is a positive real number and a, b, c are rational numbers, show that :

$$\left(\frac{x^b}{x^c}\right)^a \left(\frac{x^c}{x^a}\right)^b \left(\frac{x^a}{x^b}\right)^c = 1$$

Sol. $\left(\frac{x^b}{x^c}\right)^a . \left(\frac{x^c}{x^a}\right)^b . \left(\frac{x^a}{x^b}\right)^c = \left(x^{b-c}\right)^a . \left(x^{c-a}\right)^b . \left(x^{a-b}\right)^c = x^{ab-ac} . x^{bc-ba} . x^{ac-bc}$

$= x^{ab-ac+bc-ba+ac-bc} = x^0 = 1.$

Check Your Level

1. Find the value of the following:

(i) $81^{1/4}$ **(ii)** $64^{1/3}$ **(iii)** $32^{3/5}$ **(iv)** $27^{2/3}$

2. Simplify the following:

(i) $3^{1/3} . 3^{2/5}$ **(ii)** $\left(\frac{4}{9}\right)^{1/2}$ **(iii)** $2^{2/3}.3^{2/3}$ **(iv)** $\left(2^{1/3}\right)^3$

3. Simplified value of $2^n \times 4^n \times 8^{1-n}$

4. Find x, if $8^x = 16$

5. Find x,y,z if $15^3 x 12^2 x 16^4 = 2^x x 3^y x 5^z$

Answers

1.	**(i)**	3	**(ii)**	4	**(iii)**	8	**(iv)**	9
2.	**(i)**	$3^{11/15}$	**(ii)**	2/3	**(iii)**	$6^{2/3}$	**(iv)**	2
3.	8		**4.**	4/3	**5.**	x = 20, y = 5, z = 3		

Exercise Board Level

TYPE (I) : VERY SHORT ANSWER TYPE QUESTIONS : [01 MARK EACH]

1. Find the product of any two irrational numbers.

2. Find a rational number between $\sqrt{2}$ & $\sqrt{3}$.

3. Find the value of 1.999... in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

4. Find the number obtained on rationalising the denominator of $\frac{1}{\sqrt{7}-2}$.

5. After rationalising the denominator of $\frac{7}{3\sqrt{3}-2\sqrt{2}}$, what will be the denominator

6. Find the value of $\frac{\sqrt{32}+\sqrt{48}}{\sqrt{8}+\sqrt{12}}$.

7. Simplify $\sqrt[4]{\sqrt[3]{2^2}}$.

8. Simplify $\sqrt[4]{(81)^{-2}}$.

TYPE (II) : SHORT ANSWER TYPE QUESTIONS : [02 MARKS EACH]

9. If $\sqrt{2}$ = 1.4142, then find the value of $\sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$.

10. Find the product $\sqrt[3]{2}.\sqrt[4]{2}.\sqrt[12]{32}$.

11. Find the value of $(256)^{0.16} \times (256)^{0.09}$.

12. State whether the following statements are true or false? Justify your answer.

(i) $\frac{\sqrt{2}}{3}$ is a rational number.

(ii) There are infinitely many integers between any two integers.

(iii) Number of rational numbers between 15 and 18 is finite.

(iv) There are numbers which cannot be written in the form $\frac{p}{q}$, $q \neq 0$, p, q both are integers.

(v) The square of an irrational number is always rational.

(vi) $\frac{\sqrt{12}}{\sqrt{3}}$ is not a rational number as $\sqrt{12}$ and $\sqrt{3}$ are not integers.

(vii) $\frac{\sqrt{15}}{\sqrt{3}}$ is written in the form $\frac{p}{q}$, $q \neq 0$ and so it is a rational number.

13. Locate $\sqrt{13}$ on the number line.

14. Express $0.12\overline{3}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

15. Find the value of a in the following : $\frac{6}{3\sqrt{2}-2\sqrt{3}} = 3\sqrt{2}-a\sqrt{3}$

16. Simplify : $\left[5\left(8^{\frac{1}{3}}+27^{\frac{1}{3}}\right)^{3}\right]^{\frac{1}{4}}$.

17. Represent the following numbers on the number line :

$7, 7.2, \frac{-3}{2}, \frac{-12}{5}$

TYPE (III) : LONG ANSWER TYPE QUESTIONS : [03 MARK EACH]

18. Find three rational numbers between

(i) -1 and -2 **(ii)** 0.1 and 0.11 **(iii)** $\frac{5}{7}$ and $\frac{6}{7}$ **(iv)** $\frac{1}{4}$ and $\frac{1}{5}$

19. Insert a rational number and an irrational number between the following :

(i) 2 and 3 **(ii)** $\frac{1}{3}$ and $\frac{1}{2}$ **(iii)** $\frac{-2}{5}$ and $\frac{1}{2}$ **(iv)** $\sqrt{2}$ and $\sqrt{3}$

(v) 0.0001 and 0.001

20. Locate $\sqrt{5}$, $\sqrt{10}$ and $\sqrt{17}$ on the number line.

21. Represent geometrically the following numbers on the number line :

(i) $\sqrt{4.5}$ **(ii)** $\sqrt{2.3}$

22. Simplify the following :

(i) $\sqrt{45}-3\sqrt{20}+4\sqrt{5}$ **(ii)** $\frac{\sqrt{24}}{8}+\frac{\sqrt{54}}{9}$ **(iii)** $\sqrt[4]{12}\times\sqrt[7]{6}$

(iv) $4\sqrt{28}\div3\sqrt{7}\div\sqrt[3]{7}$ **(v)** $3\sqrt{3}+2\sqrt{27}+\frac{7}{\sqrt{3}}$ **(vi)** $\left(\sqrt{3}-\sqrt{2}\right)^2$

(vii) $\sqrt[4]{81}-8\sqrt[3]{216}+15\sqrt[5]{32}+\sqrt{225}$ **(viii)** $\frac{3}{\sqrt{8}}+\frac{1}{\sqrt{2}}$ **(ix)** $\frac{2\sqrt{3}}{3}-\frac{\sqrt{3}}{6}$

23. Rationalise the denominator of the following :

(i) $\frac{2}{3\sqrt{3}}$ **(ii)** $\frac{\sqrt{40}}{\sqrt{3}}$ **(iii)** $\frac{3+\sqrt{2}}{4\sqrt{2}}$ **(iv)** $\frac{16}{\sqrt{41}-5}$

(v) $\frac{2+\sqrt{3}}{2-\sqrt{3}}$ **(vi)** $\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$ **(vii)** $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ **(viii)** $\frac{3\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

(ix) $\frac{4\sqrt{3}+5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

24. Find the values of a and b in each of the following :

(i) $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = a-6\sqrt{3}$ **(ii)** $\frac{3-\sqrt{5}}{3+2\sqrt{5}} = a\sqrt{5}-\frac{19}{11}$

(iii) $\frac{\sqrt{2}+\sqrt{3}}{3\sqrt{2}-2\sqrt{3}} = 2-b\sqrt{6}$ **(iv)** $\frac{7+\sqrt{5}}{7-\sqrt{5}}-\frac{7-\sqrt{5}}{7+\sqrt{5}} = a+\frac{7}{11}\sqrt{5}b$

25. If a = 2 + $\sqrt{3}$, then find the value of $a - \frac{1}{a}$.

26. If $a = 5 + 2\sqrt{6}$ and $b = \frac{1}{a}$, then what will be the value of $a^2 + b^2$?

27. If $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, then find the value of $\frac{4}{3\sqrt{3} - 2\sqrt{2}} + \frac{3}{3\sqrt{3} + 2\sqrt{2}}$.

TYPE (IV): VERY LONG ANSWER TYPE QUESTIONS **[04 MARK EACH]**

28. Rationalise the denominator in each of the following and hence evaluate by taking $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$ and $\sqrt{5} = 2.236$, upto three places of decimal.

(i) $\frac{4}{\sqrt{3}}$ **(ii)** $\frac{6}{\sqrt{6}}$ **(iii)** $\frac{\sqrt{10} - \sqrt{5}}{2}$

(iv) $\frac{\sqrt{2}}{2 + \sqrt{2}}$ **(v)** $\frac{1}{\sqrt{3} + \sqrt{2}}$

29. Simplify :

(i) $\left(1^3 + 2^3 + 3^3\right)^{\frac{1}{2}}$ **(ii)** $\left(\frac{3}{5}\right)^4 \left(\frac{8}{5}\right)^{-12} \left(\frac{32}{5}\right)^6$ **(iii)** $\left(\frac{1}{27}\right)^{\frac{-2}{3}}$

(iv) $\left[\left((625)^{-\frac{1}{2}}\right)^{-\frac{1}{4}}\right]^2$ **(v)** $\frac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{\frac{1}{6}} \times 3^{-\frac{2}{3}}}$ **(vi)** $64^{-\frac{1}{3}}\left[64^{\frac{1}{3}} - 64^{\frac{2}{3}}\right]$

(vii) $\frac{8^{\frac{1}{3}} \times 16^{\frac{1}{3}}}{32^{-\frac{1}{3}}}$

30. If $a = \frac{3 + \sqrt{5}}{2}$, then find the value of $a^2 + \frac{1}{a^2}$.

31. If $x = \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} - \sqrt{2}}$ and $y = \frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$, then find the value of $x^2 + y^2$.

32. Express $0.6 + 0.\bar{7} + 0.4\bar{7}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

33. Simplify : $\frac{7\sqrt{3}}{\sqrt{10} + \sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6} + \sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15} + 3\sqrt{2}}$.

34. Find the value of $\frac{4}{(216)^{-\frac{2}{3}}} + \frac{1}{(256)^{-\frac{3}{4}}} + \frac{2}{(243)^{-\frac{1}{5}}}$.

Exercise-1

SUBJECTIVE QUESTIONS

Subjective Easy, only learning value problems

Section (A) : Introduction to number system & rational numbers

A.1 Express $\frac{2}{11}$ in decimal form.

A.2 Give three rational numbers between – 2 and – 1.

A.3 Find five rational numbers between $\frac{3}{5}$ and $\frac{4}{5}$.

A.4 Express the following in the form of p/q.

(i) $0.\overline{37}$ **(ii)** $43.\overline{5}4$ **(iii)** $5.3\overline{245}$ **(iv)** $4.62\overline{1}$

Section (B) : Irrational numbers

B.1 Write three irrational number between $\sqrt{3}$ and $\sqrt{5}$.

B.2 Find three different irrational numbers between $\frac{5}{7}$ and $\frac{9}{11}$.

B.3 Give one example where the product of two different irrational number is rational.

B.4 Prove that $7+\sqrt{3}$ is an irrational number.

B.5 Represent $\sqrt{4}$, $\sqrt{5}$, $\sqrt{10}$ on the real number line.

B.6 Represent $\sqrt{8.3}$ on the number line.

Section (C) : Surds and their application

C.1 Multiply $3\sqrt{28}$ by $2\sqrt{7}$

C.2 Find the value of $2\sqrt{5}+3\sqrt{5}$.

C.3 What is the square root of the number 0.04 in fraction form?

C.4 Simplify the expression $\frac{3}{\sqrt{48}-\sqrt{75}}$.

C.5 Find the value of $\frac{6}{\sqrt{5}-\sqrt{3}}$, if being given that $\sqrt{3}$ = 1.732 and $\sqrt{5}$ = 2.236.

C.6 Multiply $\sqrt{27a^3b^2c^4} \times \sqrt[3]{128a^7b^9c^2} \times \sqrt[6]{729ab^{12}c^2}$.

C.7 Simplify :

(i) $\frac{3\sqrt{2}}{\sqrt{6}-\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}-\sqrt{2}}+\frac{2\sqrt{3}}{\sqrt{6}+2}$ **(ii)** $\frac{7\sqrt{3}-5\sqrt{2}}{\sqrt{48}+\sqrt{18}}$

C.8 Find the value of a and b :

(i) $\frac{\sqrt{11}-\sqrt{7}}{\sqrt{11}+\sqrt{7}}=a-b\sqrt{77}$ **(ii)** $\frac{3+\sqrt{6}}{\sqrt{3}+\sqrt{2}}=a+b\sqrt{3}$

C.9 If $x=\frac{\sqrt{3}+1}{2}$, find the value of $4x^3+2x^2-8x+7$.

C.10 Prove that : $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}=5$

C.11 Arrange the following surds in ascending order of magnitude : $\sqrt[3]{2},\ \sqrt[6]{3},\ \sqrt[9]{4}$.

C.12 If $x = 2+\sqrt{3}$, find the value of $x^3+\frac{1}{x^3}$.

Section (D) : Exponents

D.1 Find the value of x : $5^{x-2}\times 3^{2x-3}=135$.

D.2 Evaluate : $\frac{3^{-3}\times 6^2\times\sqrt{98}}{5^2\times\sqrt[3]{1/25}\times(15)^{-4/3}\times 3^{1/3}}$.

D.3 Simplify : $\frac{1}{1+x^{b-a}+x^{c-a}}+\frac{1}{1+x^{a-b}+x^{c-b}}+\frac{1}{1+x^{a-c}+x^{b-c}}$.

D.4 If $\frac{9^n\times 3^2\times\left[3^{-n/2}\right]^{-2}-(27)^n}{3^{3m}\times 2^3}=\frac{1}{27}$, then prove that $m-n=1$.

D.5 If $a^x=b^y=c^z$ and $b^2=ac$, then prove that $\frac{1}{x}+\frac{1}{z}=\frac{2}{y}$.

OBJECTIVE QUESTIONS

Single Choice Objective, straight concept/formula oriented

Section (A) : Introduction to number system & rational numbers

A.1 The decimal representation of $\frac{27}{400}$ is :

(A) Terminating (B) Non terminating recurring
(C) Non terminating non recurring (D) None of these

A.2 $2.2\overline{34}$ is :

(A) Non-terminating only (B) Non-repeating only
(C) Non-terminating and repeating (D) Non-terminating and non-repeating

A.3 How many rational numbers exist between any two distinct rational numbers?

(A) 2 (B) 3 (C) 11 (D) Infinite

A.4 The rational form of $2.74\overline{35}$ is :

(A) $\frac{27161}{9999}$ (B) $\frac{27}{99}$ (C) $\frac{27161}{9900}$ (D) $\frac{27161}{9000}$

Section (B) : Irrational numbers

B.1 Which of the following number is irrational ?

(A) $\sqrt{16} - 4$ (B) $(3 - \sqrt{3})(3 + \sqrt{3})$ (C) $\sqrt{5} + 3$ (D) $-\sqrt{25}$

B.2 The decimal expansion of $\sqrt{2}$ is :

(A) Finite decimal (B) 1.4121
(C) Non-terminating recurring (D) Non - terminating non recurring

B.3 The product of a non -zero rational number with an irrational number is :

(A) Irrational number (B) Rational number (C) Whole number (D) Natural number

B.4 The number $\frac{3-\sqrt{3}}{3+\sqrt{3}}$ is :

(A) rational (B) irrational (C) both (D) can't say

B.5 Every point on a number line represents :

(A) A natural number (B) A real number (C) A rational number (D) A irrational number

B.6 A rational number lying between $\sqrt{2}$ and $\sqrt{3}$ is :

(A) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (B) $\sqrt{6}$ (C) 1.6 (D) 1.9

Section (C) : Surds and their application

C.1 If $\sqrt{3} = 1.732$ and $\sqrt{2} = 1.414$, the value of $\frac{1}{\sqrt{3}-\sqrt{2}}$ is :

(A) 0.318 (B) 3.146 (C) $\frac{1}{3.146}$ (D) $\sqrt{1.732} - \sqrt{1.414}$

C.2 If $x = 2 + \sqrt{3}$, then $\left(x + \frac{1}{x}\right)$ equals to :

(A) $-2\sqrt{3}$ (B) 2 (C) 4 (D) $4 - 2\sqrt{3}$

C.3 The exponential form of $\sqrt{\sqrt{2}\sqrt{3}}$ is :

(A) $6^{1/2}$ (B) $6^{1/3}$ (C) $6^{1/4}$ (D) 6

C.4 The simplest rationalisation factor of $\sqrt{50}$ is :

(A) 5 (B) $\sqrt{2}$ (C) 50 (D) $\sqrt{50}$

C.5 If $x = 3 + \sqrt{8}$ and $y = 3 - \sqrt{8}$ then $\frac{1}{x^2} + \frac{1}{y^2} =$

(A) -34 (B) 34 (C) $12\sqrt{8}$ (D) $-12\sqrt{8}$

C.6 If $\frac{3+\sqrt{7}}{3-\sqrt{7}} = a + b\sqrt{7}$ then (a, b) =

(A) (8, – 3) (B) (– 8, – 3) (C) (– 8, 3) (D) (8, 3)

C.7 Which one is greatest in the following :

(A) $\sqrt{2}$ (B) $\sqrt[3]{3}$ (C) $\sqrt[3]{4}$ (D) $\sqrt[3]{2}$

C.8 The value of $\sqrt[5]{(32)^{-3}}$ is :

(A) 1/8 (B) 1/16 (C) 1/32 (D) None of these

C.9 If $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ the value of $x^2 + xy + y^2$ is :

(A) 99 (B) 100 (C) 1 (D) 0

C.10 Simplify : $\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{2}} - \frac{3}{\sqrt{5}+\sqrt{2}}$.

(A) 1 (B) 0 (C) 10 (D) 100

C.11 Which of the following is smallest :

(A) $\sqrt[4]{5}$ (B) $\sqrt[5]{4}$ (C) $\sqrt{4}$ (D) $\sqrt{3}$

C.12 The product of $\sqrt{3}$ and $\sqrt[3]{5}$ is :

(A) $\sqrt[6]{375}$ (B) $\sqrt[6]{675}$ (C) $\sqrt[6]{575}$ (D) $\sqrt[6]{475}$

C.13 The value of $\sqrt{20} \times \sqrt{5}$ is :

(A) 10 (B) $2\sqrt{5}$ (C) $20\sqrt{5}$ (D) $4\sqrt{5}$

C.14 $\sqrt[3]{\frac{54}{250}}$ equals :

(A) $\frac{9}{25}$ (B) $\frac{3}{5}$ (C) $\frac{27}{125}$ (D) $\frac{\sqrt[3]{2}}{5}$

Section (D) : Exponents

D.1 The value of x, if $5^{x-3} . 3^{2x-8} = 225$, is :

(A) 1 (B) 2 (C) 3 (D) 5

D-2. $\frac{3^{5x} \times (81)^2 \times 6561}{3^{2x}} = 3^7$, then

(A) x = – 2 (B) x = – 3 (C) x = – 1 (D) x = 0

D-3. $3^n \times 9^n \times 27^{1-n} =$

(A) 9 (B) 27 (C) 3 (D) $\frac{1}{3}$

D-4. $(3^{-1} + 5^{-1} + 2^{-1})^{-1} =$

(A) $\frac{29}{30}$ (B) $\frac{31}{29}$ (C) $\frac{30}{31}$ (D) None

D-5. If $2^x = 4^y = 8^z$, then find x : y : z.

(A) 1 : 2 : 3 (B) 3 : 2 : 1 (C) 2 : 3 : 1 (D) 6 : 3 : 2

Exercise-2

OBJECTIVE QUESTIONS

1. The digit at the 100th place in the decimal representation of $\frac{6}{7}$, is :

(A) 1 (B) 2 (C) 4 (D) 5

2. xy is a number that is divided by ab where xy < ab and gives a result 0.xyxyxy... then ab equals :

(A) 11 (B) 33 (C) 99 (D) 66

3. When the repeating decimal 0.45454545....... is written in simplest fractional form, the sum of the numerator and denominator is :

(A) 5 (B) 11 (C) 55 (D) 16

4. If $\sqrt{9-(n-2)^2}$ is a real number, then the number of integral values of n is :

(A) 3 (B) 5 (C) 7 (D) Infinitely many

5. If x is a positive integer less than 100, then the number of x which make $\sqrt{1+2+3+4+x}$ an integer is:

(A) 6 (B) 7 (C) 8 (D) 9

6. If $\frac{\sqrt{954+\sqrt{\sqrt{484}+\sqrt{704+\sqrt{625}}}}}{0.00155} = \frac{\sqrt{0.0004}}{(0.009)^2.x}$. Then approximate value of x is

(A) 123.456 (B) 0.01234 (C) 12.34 (D) 12345.67

7. If n is a perfect square, then the next perfect square greater than n is :

(A) $n^2 + 1$ (B) $n^2 + n$ (C) $n + 2\sqrt{n} + 1$ (D) $2n + 1$

8. If $x+\sqrt{x+\sqrt{x+\sqrt{x+.......}}} = 2$ then x equals

(A) $2-\sqrt{2}$ (B) $2+\sqrt{2}$ (C) $2\pm\sqrt{2}$ (D) $2-\sqrt{3}$

9. The four digit number 2652 is such that any two consecutive digits from it make a multiple of 13. Another number N has this same property, is 100 digits long, and begins in a 9. The last digit of N, is

(A) 2 (B) 3 (C) 6 (D) 9

10. The expression

$$\left(\frac{1}{2}+\frac{1}{3}+.......+\frac{1}{2008}\right)\left(1+\frac{1}{2}+\frac{1}{3}+.......+\frac{1}{2007}\right)-\left(1+\frac{1}{2}+\frac{1}{3}+.......+\frac{1}{2008}\right)\left(\frac{1}{2}+\frac{1}{3}+.......+\frac{1}{2007}\right)$$

simplifies to :

(A) 0 (B) $\frac{1}{2007}$ (C) $\frac{1}{2008}$ (D) $\frac{2}{2007}$

11. If $\dfrac{1}{\dfrac{1}{\dfrac{1}{\frac{1}{x}+\frac{1}{2}}+\dfrac{1}{\frac{1}{x}+\frac{1}{2}}}+\dfrac{1}{\dfrac{1}{\frac{1}{x}+\frac{1}{2}}+\dfrac{1}{\frac{1}{x}+\frac{1}{2}}}} = \frac{x}{36}$, then x is equal to :

(A) 70 (B) 72 (C) 36 (D) 68

12. The value of $\left(1-\frac{1}{3}\right)^2\left(1-\frac{1}{4}\right)^2\left(1-\frac{1}{5}\right)^2$ .... $\left(1-\frac{1}{n}\right)^2$ is equal to :

(A) $\left(\frac{1}{n}\right)^2$ (B) $\left(\frac{2}{n}\right)^2$ (C) $\left(\frac{3}{n}\right)^2$ (D) $\left(\frac{4}{n}\right)^2$

13. If x = $(7 + 4\sqrt{3})$, then the value of $\sqrt{x} + \frac{1}{\sqrt{x}}$ is :

(A) 8 (B) 6 (C) 5 (D) 4

14. If $x = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ and $y = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ the value of $x^2 + xy + y^2$ is :

(A) 99 (B) 100 (C) 1 (D) 0

15. The value of the expression $\sqrt{\frac{1}{\sqrt{2}+1}+\frac{1}{\sqrt{3}+\sqrt{2}}+\frac{1}{\sqrt{4}+\sqrt{3}}+\text{.....upto 99 terms}}$ is equal to

(A) 9 (B) 3 (C) 1 (D) 0

16. The simplest rationalising factor of $\sqrt[4]{48}$ is :

(A) $\sqrt[4]{9}$ (B) $\sqrt[4]{27}$ (C) $\sqrt[3]{9}$ (D) None of these

17. Let r and s be integer. The $\frac{6^{r+s} \times 12^{r-s}}{8^r \times 9^{r+2s}}$ is an integer if :

(A) $r + s \le 0$ (B) $s \le 0$ (C) $r \le 0$ (D) $r \ge s$

18. If $2^a > 4^c$ and $3^b > 9^a$ and a, b, c all positive, then :

(A) c < a < b (B) b < c < a (C) c < b < a (D) a < b < c

19. If $2^{2008} - 2^{2007} - 2^{2006} + 2^{2005} = k \cdot 2^{2005}$ then the value of k is equal to :

(A) 2 (B) 3 (C) 4 (D) 5

20. If $2^x = 3^y = 6^{-z}$, then is equal to :

(A) 0 (B) 1 (C) $\frac{3}{2}$ (D) $-\frac{1}{2}$

21. If $4^x - 4^{x-1} = 24$, then the value of $(2x)^x$ equals :

(A) $5\sqrt{5}$ (B) $\sqrt{5}$ (C) $25\sqrt{5}$ (D) 25

22. $2^{73} - 2^{72} - 2^{71}$ is the same as :

(A) 2^{69} (B) 2^{70} (C) 2^{71} (D) 2^{72}

23. If x = – 0.5 then which one of the following has the smallest value :

(A) $2^{\frac{1}{x}}$ (B) $\frac{1}{x}$ (C) 2^x (D) $\frac{1}{\sqrt{-x}}$

Exercise-3

NTSE PROBLEMS (PREVIOUS YEARS)

1. If $2^x = 4^y = 8^z$ and $\frac{1}{2x} + \frac{1}{4y} + \frac{1}{6z} = \frac{24}{7}$, then the value of z is **(Rajasthan NTSE Stage-1 2005)**

(A) $\frac{7}{16}$ (B) $\frac{7}{32}$ (C) $\frac{7}{48}$ (D) $\frac{7}{64}$

2. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$ then the value of x is - **(Rajasthan NTSE Stage-1 2005)**

(A) 1 (B) 2 (C) 3 (D) 4

3. If $a^x = b$, $b^y = c$ and $c^z = a$, then value of xyz is **(Rajasthan NTSE Stage-1 2007)**

(A) 1 (B) 0 (C) –1 (D) a + b + c.

4. An equivalent expression of $\frac{5}{\sqrt{3}-\sqrt{5}}$ after rationlizing the denominator is

(Rajasthan NTSE Stage-1 2013)

(A) $\left(\frac{5}{2}\right)(\sqrt{3} + \sqrt{5})$ (B) $\left(-\frac{5}{2}\right)(\sqrt{3} + \sqrt{5})$ (C) $\left(\frac{5}{2}\right)(\sqrt{3} - \sqrt{5})$ (D) $\left(-\frac{5}{2}\right)(\sqrt{3} - \sqrt{5})$

5. Value of $\frac{2^{100}}{2}$ is : **(Rajasthan NTSE Stage-1 2013)**

(A) 1 (B) 50^{100} (C) 2^{50} (D) 2^{99}

6. If $x^{p^q} = (x^p)^q$, then p = **(Haryana NTSE Stage-1 2013)**

(A) $q^{\frac{1}{q}}$ (B) 1 (C) q^q (D) $q^{\frac{1}{q-1}}$

7. If $a^x = b$, $b^y = c$ and $c^z = a$, then the value of $x^2y^2z^2$ is ....

[Madhya Pradesh NTSE Stage-1 2013]

(A) $a^2b^2c^2$ (B) 1 (C) 4 (D) $\frac{1}{a^2b^2c^2}$

8. H.C.F. (28, 35, 91) = ... **[Gujarat NTSE Stage-1 2013]**

(A) 1 (B) 5 (C) 7 (D) 14

9. Which of the following time expressions is right for the fraction $\frac{1}{4}$?

[Gujarat NTSE Stage-1 2014]

(A) 15 minute (B) 30 minute (C) 45 minute (D) 10 minute

10. Which real number lies between 2 and 2.5 **(Chandigarh NTSE Stage-1 2014)**

(A) $\sqrt{11}$ (B) $\sqrt{8}$ (C) $\sqrt[3]{7}$ (D) $\sqrt[3]{9}$

11. Of the following four numbers the largest is : **(Harayana NTSE Stage-1 2014)**

(A) 3^{210} (B) 7^{140} (C) $(17)^{105}$ (D) $(31)^{84}$

12. The rationalizing factor of $\sqrt[n]{\frac{a}{b}}$ is **(Karnataka NTSE Stage-1 2014)**

(A) $ab\sqrt[n]{\frac{a}{b}}$ (B) $\sqrt[n]{\frac{a}{b}}$ (C) $\sqrt[n]{\frac{a^{n-1}}{b^{n-1}}}$ (D) $\sqrt[n]{\frac{a^{n+1}}{b^{n+1}}}$

13. In a number is divided by 6, the remainder is 3 then what will be the remainder when the square of the same number is divided by 6 again **[Uttar Pradesh NTSE Stage-1 2014]**
(A) 0 (B) 1 (C) 2 (D) 3

14. If x = $(3+\sqrt{8})$, then $\left(x^2+\frac{1}{x^2}\right)$ will be- **[Uttar Pradesh NTSE Stage-1 2014]**
(A) 38 (B) 36 (C) 34 (D) 30

15. If $\left(\frac{a}{b}\right)^{x-1} = \left(\frac{b}{a}\right)^{x-3}$ then the value of x will be- **[Uttar Pradesh NTSE Stage-1 2014]**
(A) –1 (B) 1 (C) 2 (D) 3

16. The simplest rationalizing factor of $\sqrt[3]{72}$ is **[Madhya Pradesh NTSE Stage-1 2014]**
(A) $2^{\frac{1}{3}}$ (B) $3^{\frac{1}{3}}$ (C) $3^{\frac{1}{2}}$ (D) $2^{\frac{1}{2}}$

17. The value of $\left(\frac{x^b}{x^c}\right)^{\frac{1}{bc}} \cdot \left(\frac{x^c}{x^a}\right)^{\frac{1}{ca}} \cdot \left(\frac{x^a}{x^b}\right)^{\frac{1}{ab}}$ on simplifying is : **(Rajasthan NTSE Stage-1 2015)**
(A) x (B) 1/x (C) 1 (D) – 1

18. If x = 2 + $\sqrt{3}$ and xy = 1 then $\frac{x}{\sqrt{2}+\sqrt{x}} + \frac{y}{\sqrt{2}-\sqrt{y}}$ = ... **(Bihar NTSE Stage-1 2015)**
(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) 1 (D) None of these

19. Simple form of $\frac{\sqrt{5}-2}{\sqrt{5}+2} + \frac{\sqrt{5}+2}{\sqrt{5}-2}$ is .... **[Madhya Pradesh NTSE Stage-1 2015]**
(A) 9 + $\sqrt{5}$ (B) 18 (C) 18 + $\sqrt{5}$ (D) 9

20. If the L.C.M. of two numbers is 2520 and H.C.F. is 12.Its one number is 504, then the other number will be .... **[Madhya Pradesh NTSE Stage-1 2015]**
(A) 50 (B) 65 (C) 40 (D) 60

21. Which number is the inverse of the opposite of $-\frac{5}{8}$? **[Gujarat NTSE Stage-1 2015]**
(A) $1\frac{5}{8}$ (B) $1\frac{3}{5}$ (C) $2\frac{2}{5}$ (D) $-\frac{8}{5}$

22. If x = $\sqrt[4]{16}$ + $\sqrt[4]{625}$,then what is x = ? **[Gujarat NTSE Stage-1 2015]**
(A) 7 (B) 29 (C) 12 (D) 5

23. The simplified value of $\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}} + \frac{1}{\sqrt{2}-\sqrt{3}-\sqrt{5}}$ is **[Delhi NTSE Stage-1 2015]**
(A) 1 (B) 0 (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

24. Raj wanted to type the first 200 natural numbers, how many times does he have to press the keys **[Delhi NTSE Stage-1 2015]**
(A) 489 (B) 492 (C) 400 (D) 365

25. Which is the greatest among $\sqrt[6]{100}$, $\sqrt[3]{12}$ and $\sqrt{3}$ **[Delhi NTSE Stage-1 2015]**

(A) $\sqrt{3}$ (B) $\sqrt[6]{100}$ (C) $\sqrt[3]{12}$ (D) cannot be determined

26. Among the numbers 2^{250}, 3^{200}, 4^{150} and 5^{100}, the greatest is **(West Bengal NTSE Stage-1 2016)**

(A) 2^{250} (B) 3^{200} (C) 4^{150} (D) 5^{100}

27. What is the square root of $9 + 2\sqrt{14}$? **(Bihar NTSE Stage-1 2016)**

(A) $1 + 2\sqrt{2}$ (B) $\sqrt{2}+\sqrt{7}$ (C) $\sqrt{3}+\sqrt{6}$ (D) $\sqrt{2}+\sqrt{5}$

28. $\sqrt[3]{1-\frac{127}{343}}$ is equal to **(Bihar NTSE Stage-1 2016)**

(A) $\frac{5}{9}$ (B) $1-\frac{1}{7}$ (C) $\frac{4}{7}$ (D) $1-\frac{2}{7}$

29. What is the value of $2.\overline{6} - 1.\overline{9}$? **(Bihar NTSE Stage-1 2016)**

(A) $0.\overline{6}$ (B) $0.\overline{9}$ (C) $0.\overline{7}$ (D) 0.7

30. An equivalent expression of $\frac{5}{7+4\sqrt{5}}$ after rationalizing the denominator is **[Gujarat NTSE Stage-1 2016]**

(A) $\frac{20\sqrt{5}-35}{31}$ (B) $\frac{20\sqrt{5}-35}{129}$ (C) $\frac{35-20\sqrt{5}}{31}$ (D) $\frac{35-20\sqrt{5}}{121}$

31. Four positive integers sum to 125. If the first of these numbers is increased by 4, the second is decreased by 4. the third is multiplied by 4 and the fourth is divided by 4 then we get four equal numbers then four original integers are **[Delhi NTSE Stage-1 2016]**

(A) 16, 24, 5, 80 (B) 8, 22, 38, 57 (C) 7, 19, 46, 53 (D) 12, 28, 40, 45

32. If $a=\sqrt{6}+\sqrt{5};$, $b=\sqrt{6}-\sqrt{5}$ the find the value of $2a^2-5ab+2b^2$ **[Maharashtra NTSE Stage-1 2016]**

(A) 36 (B) 37 (C) 39 (D) 41

33. $\sqrt{m^4n^4}\times\sqrt[6]{m^2n^2}\times\sqrt[3]{m^2n^2}=(mn)^k$, then find the value of k. **[Maharashtra NTSE Stage-1 2017]**

(A) 6 (B) 3 (C) 2 (D) 1

Answer Key

Exercise Board Level

TYPE (I)

1. Rational or Irrational **2.** 1.5 **3.** 2 **4.** $\frac{\sqrt{7}+2}{3}$

5. $\frac{7(3\sqrt{3}+2\sqrt{2})}{19}$ **6.** 2 **7.** $2^{\frac{1}{6}}$ **8.** $\frac{1}{9}$

TYPE (II)

9. 0.4142 **10.** 2 **11.** 4

12. **(i)** False **(ii)** False **(iii)** False **(iv)** True

(v) False **(vi)** False **(vii)** False

13. OA = 2 units and AB = 3 units. **14.** $\frac{37}{300}$ **15.** a = – 2 **16.** 5

TYPE (III)

18. **(i)** – 1.1, – 1.2, – 1.3 **(ii)** 0.101, 0.102, 0.103

(iii) $\frac{51}{70}, \frac{52}{70}, \frac{53}{70}$ **(iv)** $\frac{17}{80}, \frac{18}{80}, \frac{19}{80}$

19. **(i)** $\frac{5}{2}$ **(ii)** $\frac{5}{12}$ **(iii)** 0 **(iv)** 1.5 **(v)** 0.0001131331333….

22. **(i)** $\sqrt{5}$ **(i)** $\frac{7\sqrt{6}}{12}$ **(iii)** $\sqrt[28]{2^{18}.3^{11}}$ **(iv)** $\frac{8}{3\sqrt[3]{7}}$

(v) $\frac{34}{\sqrt{3}}$ **(vi)** $5-2\sqrt{6}$ **(vii)** 0 **(viii)** $\frac{5\sqrt{2}}{4}$

(ix) $\frac{\sqrt{3}}{2}$

23. **(i)** $\frac{2\sqrt{3}}{9}$ **(ii)** $\frac{2\sqrt{30}}{3}$ **(iii)** $\frac{3\sqrt{2}+2}{8}$ **(iv)** $\sqrt{41}+5$

(v) $7+4\sqrt{3}$ **(vi)** $3\sqrt{2}-2\sqrt{3}$ **(vii)** $5+2\sqrt{6}$ **(viii)** $9+2\sqrt{15}$

(ix) $\frac{9+4\sqrt{6}}{15}$

24. **(i)** a = 11 **(ii)** a = $\frac{9}{11}$ **(iii)** $\frac{-5}{6}$ **(iv)** a = 0 , b = 1

25. $2\sqrt{3}$ **26.** 98 **27.** 2.063

TYPE (IV)

28. **(i)** 2.309 **(ii)** 2.449 **(iii)** 0.463 **(iv)** 0.414

(v) 0.318

29. **(i)** 6 **(ii)** $\frac{2025}{64}$ **(iii)** 9 **(iv)** 5

(v) $3^{\frac{-1}{3}}$ **(vi)** – 3 **(vii)** 16

30. 7 **31.** 98 **32.** $\frac{167}{90}$ **33.** 1 **34.** 214

Exercise-1

SUBJECTIVE QUESTIONS

Section (A)

A.1 0.1818 **A.2** $-\frac{7}{4}, -\frac{3}{2}, -\frac{5}{4}$ **A.3** $\frac{19}{30}, \frac{2}{3}, \frac{7}{10}, \frac{11}{15}, \frac{23}{30}$

A.4 **(i)** $\frac{37}{99}$ **(ii)** $\frac{479}{11}$ **(iii)** $\frac{53192}{9990}$ **(iv)** $\frac{4159}{900}$

Section (B)

B.1 1.801001....,1.901001......,2.01001........

B.2 0.7507500750007......,0.7670767007670007..... , and 0.808008000800008.....

B.3 $(3 + \sqrt{2})(3 - \sqrt{2})$

Section (C)

C.1 84 **C.2** $5\sqrt{5}$ **C.3** $\frac{1}{5}$ **C.4** $-\sqrt{3}$

C.5 11.904. **C.6** $36\, a^4 b^6 c^3 \sqrt[6]{108}$ **C.7** **(i)** 0 **(ii)** $\frac{114 - 41\sqrt{6}}{30}$

C.8 **(i)** $a = \frac{9}{2}$ & $b = \frac{1}{2}$. **(ii)** a = 0 and b = 1. **C.9** 10

C.11 $\sqrt[9]{4} < \sqrt[6]{3} < \sqrt[3]{2}$ **C.12** 52

Section (D)

D.1 3 **D.2** $28\sqrt{2}$ **D.3** 1

OBJECTIVE QUESTIONS

Section (A)

A.1 (A) **A.2** (C) **A.3** (D) **A.4** (C)

Section (B)

B.1 (C) **B.2** (D) **B.3** (A) **B.4** (B) **B.5** (B)

B.6 (C)

Section (C)

C.1 (B) **C.2** (C) **C.3** (C) **C.4** (B) **C.5** (B)

C.6 (D) **C.7** (C) **C.8** (A) **C.9** (A) **C.10** (B)

C.11 (B) **C.12** (B) **C.13** (A) **C.14** (B)

Section (D)

D.1 (D) **D-2.** (B) **D-3.** (B) **D-4.** (C) **D-5.** (D)

Exercise-2

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	A	C	D	C	B	B	C	A	D	C	A	B	D	A	B
Ques.	16	17	18	19	20	21	22	23							
Ans.	B	B	A	B	A	C	C	B							

Exercise-3

Ques.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	C	B	A	B	D	D	B	C	A	D	C	C	D	C	C
Ques.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	B	C	A	B	D	B	A	D	B	C	B	B	B	A	A
Ques.	31	32	33												
Ans.	A	C	B												